作业 4 参考答案

人工智能导论课（2023 春季学期）

编辑于 2023/5/23

1. 信息价值

1)

$$\begin{aligned}\text{MEU}(\emptyset) &= \max_a \sum_w P(w)U(w,a) \\ &= \max_a \{\sum_w P(w)U(w,a=\text{take}), \sum_w P(w)U(w,a=\text{leave})\} \\ &= \max\{20*0.5+70*0.5, 100*0.5+0\} \\ &= 50\end{aligned}$$

2)

$$\begin{aligned}P(W|F) &= \frac{P(W,F)}{P(F)} \\ &= \frac{P(W)P(F|W)}{\sum_W P(W,F)}\end{aligned}$$

P(W,F=good)	
sun	0.5*0.65=0.325
rain	0.5*0.55=0.275

P(W|F=good)	
sun	0.542
rain	0.458

同理，可得：

P(W|F=bad)	
sun	0.438
rain	0.562

3)

$$\begin{aligned} P(F) &= \sum_W P(W,F) \\ &= \sum_W P(W)P(F|W) \end{aligned}$$

P(W,F)	
sun,good	0.5*0.65=0.325
sun,bad	0.5*0.35=0.175
rain,good	0.5*0.55=0.275
rain,bad	0.5*0.45=0.225

P(F)	
good	0.6
bad	0.4

4)

$$\begin{aligned} \text{VPI}(F) &= \text{MEU}(F) - \text{MEU}(\emptyset) \\ \text{MEU}(F) &= \sum_f P(f)\text{MEU}(f) \\ \text{MEU}(F = good) &= \max_a \sum_w P(w|F = good)U(w,a) \\ &= \max\{\sum_w P(w|F = good)U(w, a = take), \sum_w P(w|F = good)U(w, a = leave)\} \\ &= \max\{0.542 * 20 + 0.458 * 70, 0.542 * 100 + 0\} \\ &= \max\{42.9, 54.2\} \\ &= 54.2 \end{aligned}$$

$$\begin{aligned} \text{MEU}(F = bad) &= \max_a \sum_w P(w|F = bad)U(w,a) \\ &= \max\{\sum_w P(w|F = bad)U(w, a = take), \sum_w P(w|F = bad)U(w, a = leave)\} \\ &= \max\{0.438 * 20 + 0.562 * 70, 0.438 * 100 + 0\} \\ &= \max\{48.1, 43.8\} \\ &= 48.1 \end{aligned}$$

$\text{MEU}(F) = 0.6 * 54.2 + 0.4 * 48.1 = 51.76$

$\text{VPI}(F) = 51.76 - 50 = 1.76$

2. HMM

1）

$$\begin{aligned} P(X_2, E_1 = A) &= \sum_{X_1} P(X_2, X_1, E_1 = A) \\ &= \sum_{X_1} P(X_2|X_1)P(X_1, E_1 = A) \\ &= \sum_{X_1} P(X_2|X_1)P(E_1|X_1)P(X_1) \end{aligned}$$

$P(X_1, E_1 = A)$	
0	0.2*0.8=0.16
1	0.8*0.4=0.32

$P(X_2, X_1, E_1 = A)$	
0,0	0.16*0.3=0.048
0,1	0.32*0.8=0.256
1,0	0.16*0.7=0.112
1,1	0.32*0.2=0.064

$P(X_2, E_1 = A)$	
0	0.048+0.256=0.304
1	0.112+0.064=0.176

2）

$$\begin{aligned} P(X_2, E_1 = A, E_2 = B) =& P(X_2, E_1 = A)P(E_2 = B|X_2, E_1 = A) \\ =& P(X_2, E_1 = A)P(E_2 = B|X_2) \text{ 条件独立性} \end{aligned}$$

$P(X_2, E_1 = A, E_2 = B)$	
0	0.304*0.2=0.0608
1	0.176*0.6=0.1056

$P(X_2\|E_1 = A, E_2 = B)$	
0	0.365
1	0.635

3. 粒子滤波

1)

粒子的权值由 $P(e_i|x_i)$ 来决定，即 P1 的权值是 $P(E_1 = A|X_1 = 0) = 0.8$，P2 的权值是 $P(E_1 = A|X_1 = 1) = 0.4$。

2)

X_1	权值	正规化后的权值
0	0.8	0.8/(0.8+0.4)=0.667
1	0.4	0.333

在此分布上进行类似于贝叶斯网络里的先验采样，给定随机数 {0.23，0.06}，得到两个粒子分别是 $P1 = 0, P2 = 0$。

3)

时间向前推移一步，即在概率分布 $P(X_2|X_1 = x_1)$ 上进行随机采样。根据 2）中的结果，P1，P2 都需要在 $P(X_2|X_1 = 0)$ 上进行采样。给定各自相应的随机数，看落在哪一个状态值的区间，即返回相应的状态值。

$P(X_2|X_1 = 0)$	
0	0.3
1	0.7

P1=1，P2=0.

4)

P1 的权值是 $P(E_2 = B|X_2 = 1) = 0.6$，P2 的权值是 $P(E_2 = B|X_2 = 0) = 0.2$.

5)

X_2	权值	正规化后的权值
0	0.2	0.2/(0.2+0.6)=0.25
1	0.6	0.75

给定各自相应的随机数，在以上的概率分布上进行随机采样的结果是：P1=0，P2=1.

6)

给定当前的两个粒子（P1，P2），估计此时变量 X 的后验分布是：

$P(X_2|E_1 = A, E_2 = B)$	
0	1/2
1	1/2